A Relationship that Expresses an Electron's Relativistic Mass in Terms of its Relativistic Wavelength

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Background

Physicist de Broglie showed that if it was possible for light to have momentum, then it was possible for particles like electrons to have a wavelength λ. His formula is simple:

$$\lambda = h/p,$$

where $h = 6.626 \times 10^{-34}$ J·s is Planck's constant and p is an electron's momentum given by the formula $p = mv$. In this case, m is the electron relativistic mass and v is the electron speed.

Einstein showed how the speed of an object affects its mass. In fact, as the electron speed increases, the electron mass also increases. This effect is unnoticeable for small speeds, but as speed increases, the effect cannot be ignored, especially in the case where the electron speed v approaches the speed of light c.

Description of the Problem and Solution

The goal of this short note is to determine an expression for the relativistic kinetic energy of an electron in terms of its momentum p and relativistic mass m. Then we will use this formula so that we can express its relativistic mass m in terms of its relativistic wavelength λ.

It is known that the total energy of an electron is given by:

$$E = mc^2 = [m_0/(1 - v^2/c^2)^{1/2}]c^2, \qquad (1)$$

where m_0 is the electron rest mass. From this correlation, we obtain:

$$E(1 - v^2/c^2)^{1/2} = m_0c^2,$$
$$[E(1 - v^2/c^2)^{1/2}]^2 = (m_0c^2)^2,$$

or

$$E^2 - (E^2/c^2)v^2 = E_0^2. \qquad (2)$$

An electron's momentum is given by:

$$p = mv = [m_0/(1 - v^2/c^2)^{1/2}]\, v,$$
$$p/v = m_0/(1 - v^2/c^2)^{1/2},$$

or

$$p^2/v^2 = [m_0/(1 - v^2/c^2)^{1/2}]^2. \qquad (3)$$

From (1), we obtain:

$$E/c^2 = m_0/(1 - v^2/c^2)^{1/2}$$

or $$E^2/c^4 = [m_0/(1 - v^2/c^2)^{1/2}]^2. \quad (4)$$

Equating (4) to (3), we get:

$$E^2/c^4 = p^2/v^2$$

or $$E^2/c^2 = p^2c^2/v^2. \quad (5)$$

Substituting (5) into (2), we obtain:

$$E^2 - (p^2c^2/v^2)\, v^2 = E_0^2,$$
$$E^2 - p^2c^2 = E_0^2,$$
$$E^2 - E_0^2 = p^2c^2,$$
$$p^2 = (E^2 - E_0^2)/c^2,$$

or $$p = [(E - E_0)\,(E + E_0)]^{1/2}/c. \quad (6)$$

An electron's relativistic kinetic energy is given by:

$$K = mc^2 - m_0c^2 = E - E_0. \quad (7)$$

Substituting (7) into (6), we get:

$$p = [K(E + E_0)]^{1/2}/c,$$
$$p = [K(mc^2 + m_0c^2)]^{1/2}/c,$$
$$p = [Kc^2(m + m_0)]^{1/2}/c,$$
$$p = [K(m + m_0)]^{1/2},$$

or $$K = p^2/(m + m_0). \quad (8)$$

Formula (8) expresses the relativistic kinetic energy of an electron in terms of its momentum p and relativistic mass m. Note that when $v << c$, $m \approx m_0$ and formula (8) can be written in the form $K = p^2/(2m_0)$, which determines the quantized version of the non-relativistic kinetic energy in terms of momentum.

Finally, equating (8) to (7), we obtain the relationship that we were looking for:

$$(m - m_0)\, c^2 = p^2/(m + m_0),$$
$$m^2 - m_0^2 = p^2/c^2,$$
$$m^2 = m_0^2 + p^2/c^2,$$
$$m = [m_0^2 + p^2/c^2]^{1/2},$$

or $$m = [m_0^2 + h^2/(\lambda^2c^2)]^{1/2}. \quad (9)$$

Conclusion

It seems to me that sometimes an explicit difference between $K = p^2/(2m_0)$ and correlation (8) is not established, and that $K = p^2/(2m_0)$ is used to determine values of the relativistic kinetic energy in terms of momentum. Of course, I could be wrong if that is not the case. Nevertheless, note that formula $K = p^2/(2m_0)$ is just the quantized version of the non-relativistic expression for kinetic energy in terms of momentum, while formula (8) obtained in this note provides the description of the electron relativistic kinetic energy in terms of momentum and relativistic mass. Note also that formula (9) allows us to find the value of relativistic mass m in terms of wavelength λ avoiding additional calculations.